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Analysis of Image Rotation for Aerial Remote Sensor with Off-Axis three-Reflective Optical System

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Abstract

In order to improve the dynamic state resolution of aerial remote sensor, it is indispensable to analyzing image rotation of aerial remote sensor. According to optical reflection vector theory, study on imaging character of aerial remote sensor with off-axis three-reflective optical system. At first, a simple mathematics model is abstracted from the aerial remote sensor. In the Cartesian coordinate system, compute the image plane revolution vector quantity caused by scan mirror rotating of parallactic angle and depression angle by analysis of image rotation direction for aerial remote sensor with off-axis three-reflective optical system applying coordinate conversion and geometrical optics, provide a theoretical resource for the application of the drift mechanism (eliminating image rotation mechanism). Using this method can calculate exactly image motion and image motion compensation error of image plane, and image motion quantity less than 0.003mm. In conclusion, the result of the computation improves Modulation Transfer Function (MTF) of optical system.

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Keywords: Coordinate conversion, Normal line assemble, Image rotation, Modulation Transfer Function (MTF)

1. Introduction

As a key method of acquiring ground information, aerial remote sensors have developed for several generations. It would be relative motion toward ground during flight all the while. That is a great distinguish between aerial photograph and ground photograph. That is lead to relative motion between image and CCD in time of exposure, we call this phenomenon is “Image Motion”. There is no doubt that image motion will lead to image quality fall off severely. So it had to be eliminated within a reasonable. Include mainly: aerial camera scanning mirror motion, aerial craft attitude motion, aerial craft vibration. The main reason for image motion in aerial photography is specific to the movement which can change the visual axis of aerial remote sensor. It includes the following mainly: Aerial camera scanning mirror

movement, Attitude motion of aircraft, aircraft vibration. Aerial camera scanning mirror movement and attitude motion of aircraft can also cause the problems such as image rotation in a particular way of photography. That is to say, since the aircraft attitude motion which includes pitch, yaw and roll and remote sensor's angle of depression motion in the work, that is bring about the direction of image motion vector in the focal plane coordinate system change. If you do not eliminate the image rotation, you will not get a clear image. At present, it commonly used method of eliminate the image rotation: additional optical components in front of the optical system and electronic image. According to this analysis, this paper puts forward a method of eliminating image rotation. CCD detector rotate in a reverse direction so that the system eliminates the image rotation caused scanning mirror rotating when the scan mirror rotate at the same time. There are undeniable advantages to both Optical eliminate image rotation consume time lag and structure of electronic image devices is simple relatively. Consequently, it is necessary that accurate calculating image rotation and thus to eliminating image rotation is completed successfully for improving image quality.

2. The principle and a mathematical model

2.1 Coordinate system selection

Earth-fixed axis system: origin and three axes of coordinate system fixed ground;

Body axis system: Fixed coordinate system on the plane, its origin is usually located in the aircraft gravity;

Flight-path axis system: flight-path axis system: the origin is usually fixed to the aircraft gravity, the x axis direction along the velocity of flight path, the plumb plane contains z axis and x axis, z axis perpendicular to the x axis, pointing downwards, y axis perpendicular to plumb plane, pointing to the right; flight-path axis system coincide with the body coordinate system When flight attitude angles were not considered. Usually it determined by the GPS.

Longitudinal axis (x): in the Reference plane of aircraft and point to front;

Transverse axis (y): perpendicular to Reference plane of aircraft and point to the right;

Normal axis (z): in the Reference plane of aircraft, perpendicular to longitudinal axis and point down.

2.2 Definition of aircraft attitude

It is essential for us to choose flight-path axis system and body axis system. Attitude angle is the angle of flight-path axis system relative to body axis system.

Pitch angle: the angle between the aircraft longitudinal axis and the horizontal plane, positive as aircraft head up.

Roll angle: the angle between the aircraft horizontal axis and the horizontal plane, positive as aircraft tilted to right.

Yaw angle: the angle between the longitudinal axis of aircraft and book course, the relative book course to the left is positive.

2.3 Theory of light reflection vector

The Law of reflection described by vector, a represents the incident light, N represents normal reflector, B represents Reflected light. According to object-image conjugate principle of reflector, there are:

$$\bar{B} = \begin{bmatrix} 1-2N_x^2 & -2N_xN_y & -2N_xN_z \\ -2N_xN_y & 1-2N_y^2 & -2N_yN_z \\ -2N_xN_z & -2N_yN_z & 1-2N_z^2 \end{bmatrix} \bar{A} = (E - 2NN^T)\bar{A} \quad (1)$$

In the formula: N_x, N_y, N_z are the reflector normal vector resolutions in the datum coordinate system.

The reflector rotation principle described in vector form, N_0 respectively rotate around the X, Y axis in the datum coordinate system are:

$$N_{x0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} N_0, \quad N_{y0} = \begin{bmatrix} \cos \varphi & 0 & -\sin \varphi \\ 0 & 1 & 0 \\ \sin \varphi & 0 & \cos \varphi \end{bmatrix} N_0 \quad (2)$$

In the formula: θ and φ is respectively angle which the reflector rotated around the X, Y axis in the datum coordinate system.

2.4 Establish the mathematical model of aerial remote sensor

Selecting the appropriate coordinate system in the process of establish the mathematical model will bring convenience to the calculation.

It is essential that establish body axis system XYZ in the scanning mirror. The whole model is divided into three parts: (1) Scanning mechanism, this section provides step-scanning, there are two degrees of freedom (angle of depression, parallactic angle). (2) The section of lens, including the primary mirror, secondary mirror, the third mirror and focal plane mirror, the position between them relatively static, the entire part is relatively static with the carrier. (3) Drift mechanism, this section includes the image plane revolution mechanism and the CCD detector. Drift mechanism are synchronous rotate with image during exposure. As shown in Figure 1.

3. Image rotation vector calculation

3.1 Aspherical mirror normal assemble

In a broad sense, all surfaces are aspheric surfaces except spheric surface and flat surface (which may be considered as spheric special case, radius of curvature is infinitely), it include asymmetric space curved surface, also called free-form surface.

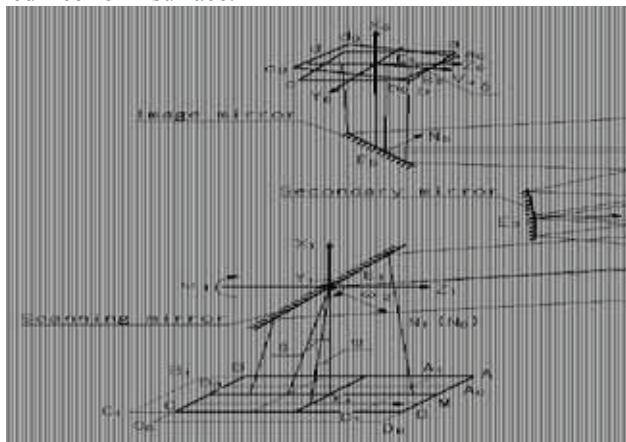


Figure 1. Mathematic model of aerial remote sensor with off-axis three-reflective optical system

In most cases, aspheric surfaces mirror photoelectric image system used axisymmetric aspheric surfaces. Revolving aspherical surface usually apply Cartesian coordinate system, it take the direction that light spreads as z-axis positive direction, the x-axis is to be radial distance away from aspherical surface, the y-axis is perpendicular to paperback and pointing outwards, with aspherical surface acme as origin of coordinate, as shown in figure 2. It's equation as follows:

$$z = \frac{cx^2}{1 + \sqrt{1 - (1+k)c^2x^2}} + \sum_{i=1}^{\infty} a_i x^{2i} \quad (3)$$

In the formula: c refers to acme curvature radius reciprocal; k refers to secondary constants; a_i refers to multinomial coefficient.

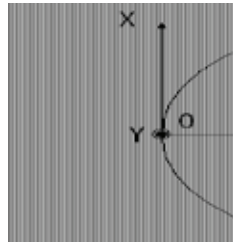


Figure 2. Aspheric Equation coordinate system

The normal vector of aspherical surface reflecting mirror is different at any point, so aspherical surface has a normal gathers, which is called normal assemble, as shown in figure 3. It converts equation (3) to curved surface equation as follows:

$$z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1+k)c^2(x^2 + y^2)}} + \sum_{i=1}^{\infty} a_i (x^2 + y^2)^i \quad (4)$$

In case that the implicit function equation is:

$$F(x, y, z) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1+k)c^2(x^2 + y^2)}} + \sum_{i=1}^{\infty} a_i (x^2 + y^2)^i - z = 0 \quad (5)$$

Because aspheric surface reflecting mirror is smooth everywhere, its curved surface equation exists continues derived function. So normal vector of aspheric surface reflecting mirror is:

$$N(F_x(x, y, z), F_y(x, y, z), -1) \quad (6)$$

It also can indicate N to vector of direction angle expression in coordinate system:

$$N = [\cos \alpha, \cos \beta, \cos \gamma] \quad (7)$$

In this equation:

$$\begin{aligned} \cos \alpha &= \frac{F_x(x, y, z)}{\sqrt{F_x(x, y, z)^2 + F_y(x, y, z)^2 + (-1)^2}} \\ \cos \beta &= \frac{F_y(x, y, z)}{\sqrt{F_x(x, y, z)^2 + F_y(x, y, z)^2 + (-1)^2}} \\ \cos \gamma &= \frac{-1}{\sqrt{F_x(x, y, z)^2 + F_y(x, y, z)^2 + (-1)^2}} \end{aligned}$$

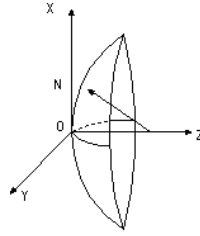


Figure 3. Normal line assemble coordinate system

3.2 Ray Tracing of off-axis three-reflective optical system

Let point $E_0(x_0, y_0, z_0, 1)$, $E_1(x_1, y_1, z_1, 1)$, $E_2(x_2, y_2, z_2, 1)$, $E_3(x_3, y_3, z_3, 1)$, $E_4(x_4, y_4, z_4, 1)$, $E_5(x_5, y_5, z_5, 1)$ are crossing points of target ray and different reflection mirror, $E_6(x_6, y_6, z_6, 1)$ is the conjugate image point of E_0 . V_k is relative speed between flight-path axis system and earth-fixed axis system. The coordinate of object point E_0 in scanning reflecting mirror coordinate system (scanning reflecting mirror coordinate system coincides with body axis system) is $(x_0, y_0, z_0, 1)$, so the vector $\overline{E_0E_1}$ of ray by E_0 which through grid origin is $(x_0, y_0, z_0, 1)$, the vector by way of scanning reflecting mirror is:

$$\overline{E_1E_2} = (E - 2N_1N_1^T)\overline{E_0E_1}$$

Normal vector of scanning reflecting mirror is:

$$N_1 = [\cos \alpha_1, \cos \beta_1, \cos \gamma_1, 1]^T$$

It can transform vector $\overline{E_1E_2}$ from coordinate system $X_1Y_1Z_1$ to coordinate system $X_2Y_2Z_2$:

$$\overline{E_1E_2}' = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & m_1 \\ 0 & 0 & 1 & n_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overline{E_1E_2} \quad (8)$$

In this equation, l_1, m_1, n_1 are respectively coordinate figure from coordinate system $X_1Y_1Z_1$ to coordinate system $X_2Y_2Z_2$, they are known quantities in system. If transform grid origin $(0,0,0,1)$ in coordinate system $X_1Y_1Z_1$ to $(l_1, m_1, n_1, 1)$ in coordinate system $X_2Y_2Z_2$, it can get reflected linear equation (in coordinate system $X_2Y_2Z_2$) by pointnorm form equation of linear line. Using analytic geometry, it can find out crossing point E_2 between $\overline{E_1E_2}'$ and curved face equation of aspheric surface. Take it to equation (6):

$$N_2 = [\cos \alpha_2, \cos \beta_2, \cos \gamma_2, 1]^T$$

In due order using mentioned above calculation process of every reflecting mirror, it can get:

$$\overline{E_5E_6}' = \begin{bmatrix} 1 & 0 & 0 & l_5 \\ 0 & 1 & 0 & m_5 \\ 0 & 0 & 1 & n_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} (E - 2N_5N_5^T) \begin{bmatrix} 1 & 0 & 0 & l_4 \\ 0 & 1 & 0 & m_4 \\ 0 & 0 & 1 & n_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} (E - 2N_4N_4^T) \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & m_3 \\ 0 & 0 & 1 & n_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ (E - 2N_3N_3^T) \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & m_2 \\ 0 & 0 & 1 & n_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} (E - 2N_2N_2^T) \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & m_1 \\ 0 & 0 & 1 & n_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} (E - 2N_1N_1^T) \overline{E_0E_1} \quad (9)$$

It can find out pointnorm form equation of linear line E_5E_6 by point E_5 and vector $\overline{E_5E_6}'$ in coordinate system $X_6Y_6Z_6$, the crossing point of the equation and flat surface X_6OZ_6 is coordinate (x_6, y_6, z_6) in coordinate system $X_6Y_6Z_6$ which is the conjugate image point E_6 with object point E_0 .

3.3 Decision of image revolution direction by scanning mirror traverses along with position angle direction

Take subastral point of remote sensor for instance, scanning mirror traverses along with position angle direction, as shown in Figure 1, space area of photograph object turn from $ABCD$ to $A_1B_1C_1D_1$. Coordinate system changes as shown in Figure 1, Euler angle which coordinate system of scanning reflecting mirror deviate from y-axis of body axis system is η , as shown in figure 4. Take $N_1 = N_{Y0}$ into equation (9), it shows that when scanning mirror traverses along with position angle direction, the problem of image revolution will not appear.

$$N_{Y0} = \begin{bmatrix} \cos \eta & 0 & -\sin \eta \\ 0 & 1 & 0 \\ \sin \eta & 0 & \cos \eta \end{bmatrix} N_0 \quad (10)$$

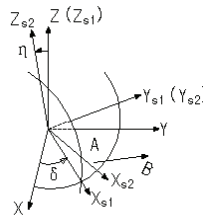


Figure 4. Revolution between body axis system and scanning mirror axis system

3.4 Decision of image revolution direction by scanning mirror traverses along with plunge angle direction

Take subastral point of remote sensor for instance, scanning mirror traverses along with plunge angle direction, as shown in Figure 1, space area of photograph object turn from $ABCD$ to $A_0B_0C_0D_0$. Coordinate system changes as shown in Figure 1, Euler angle which coordinate system of scanning reflecting mirror deviate from x-axis of body axis system is δ , as shown in figure 4. Take $N_1 = N_{X0}$ into equation (9), it shows that when scanning mirror traverses along with plunge angle direction, the problem of image revolution will appear.

$$N_{X0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & \sin \delta \\ 0 & -\sin \delta & \cos \delta \end{bmatrix} N_0 \quad (11)$$

3.5 Decision of image revolution direction by considering flight attitude of remote sensor

It includes pitch, roll and drift angle in flight process of remote sensor, these angles cause to revolution of flight-path axis system in relation to body axis system, as shown in Figure 5, vector \overline{AB} in flight-path axis system transfer vector $\overline{A'B'}$ in body axis system. Take the calculation of equation (12) into equation (9), it can know the effect of flight attitude of remote sensor to image revolution.

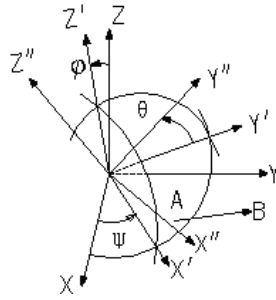


Figure 5. Revolution between flight-path axis system and body axis system

$$\overline{A'B'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \varphi & 0 & -\sin \varphi & 0 \\ 0 & 1 & 0 & 0 \\ \sin \varphi & 0 & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \psi & \sin \psi & 0 & 0 \\ -\sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overline{AB} \quad (12)$$

$$= \begin{bmatrix} \cos \varphi \cos \psi & \cos \varphi \sin \psi & -\sin \varphi & 0 \\ \sin \theta \sin \varphi \cos \psi - \cos \theta \sin \psi & \sin \theta \sin \varphi \sin \psi + \cos \theta \cos \psi & \sin \theta \cos \varphi & 0 \\ \cos \theta \sin \varphi \cos \psi + \sin \theta \sin \psi & \cos \theta \sin \varphi \sin \psi - \sin \theta \cos \psi & \cos \theta \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overline{AB}$$

Take vector $\overline{AB} = [0, 0, V_K, 1]^T$ in flight-path axis system into equation (12) and (9):

$$V_a = \begin{bmatrix} V_x \\ V_y \\ V_z \\ 1 \end{bmatrix}$$

The angle of Image surface revolution is:

$$\eta = \arctan \frac{V_y}{V_x}$$

4. Experiment of aerial remote sensor

In order to verify the correctness of the Calculus process, for example, focal length $f=1800$ mm of aerial remote sensor, primary mirror $k=-2.14574$, secondary mirror $k=-0.64136$, third mirror $k=0.226895$, pixel size is 0.009 mm. Laboratory equipment includes target generator, flight attitude turntable, aerial remote sensor, computer, and so on. The experimental data post-processed shows that the angle of image rotation is certain proportional to angle of scanning mirror turned. High-resolution images can be obtained from the CCD detector.

5. Conclusion

The above analysis shows that, the image does not produce rotation when the scan mirror rotates along the direction of parallax angle. The image does not produce rotation when the scan mirror rotates along the direction of Depression angle. the angle of image rotation is certain proportional to angle of scanning mirror turned and opposite in direction. Therefore, this method can be used to eliminate image rotation by scanning mirror rotation Synchronized with the CCD detector in the opposite direction. Furthermore, the image rotation of any two points in image space can be obtained by Vector operation about any two points in object space. By this way, the image motion caused by aircraft vibration and flight attitude which includes pitch, yaw and roll can be analyzed. Applied to the aerospace field, image

motion and image rotation due to drift and height of flight path can also be analyzed by earth core-centroid axis system and so on.

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